On the Energy Efficiency of Orthogonal Signaling

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Abstract— [1] **In this paper, transmission over the additive white Gaussian noise (AWGN) channel, and coherent and noncoherent fading channels using M-ary orthogonal frequency-shift keying (FSK) or on-off frequency-shift keying (OOFSK) is considered. The receiver is assumed to perform hard-decision detection. In this setting, energy required to reliably send one bit of information is investigated. It is shown that for fixed M and duty cycle, bit energy requirements grow without bound as the signal-to-noise ratio (SNR) vanishes. The minimum bit energy values are numerically obtained for different values of M and the duty cycle. The impact of fading on the energy efficiency is identified. Requirements to approach the minimum bit energy of -1.59 dB are determined.**

I. INTRODUCTION

Energy efficient transmission is of paramount importance in many communication systems and particularly in mobile wireless systems due to the scarcity of energy resources. Energy efficiency can be measured by the energy required to send one information bit reliably. It is well-known that for Gaussian channels subject to average input power constraints, the minimum received bit energy normalized by the noise spectral level is $\frac{E_b}{N_0}{}_{\min} = -1.59$ dB regardless of the availability of channel side information (CSI) at the receiver (see e.g., [1] – [7]). Golay [1] showed that this minimum bit energy can be achieved in the additive white Gaussian noise (AWGN) channel by pulse position modulation (PPM) with vanishing duty cycle when the receiver employs threshold detection. Indeed, Turin [2] proved that any orthogonal M-ary modulation scheme with envelope detection at the receiver achieves the normalized bit energy of -1.59 dB in the AWGN channel as $M \to \infty$. It is further shown in [3] and [4] that M-ary orthogonal frequency-shift keying (FSK) achieves this minimum bit energy asymptotically as $M \to \infty$ also in noncoherent fading channels where neither the receiver nor the transmitter knows the fading coefficients. As also well-known by now in the digital communications literature [14], these results are shown by proving that the error probabilities of orthogonal signaling can be made arbitrarily small as $M \to \infty$ as long as the normalized bit energy (or equivalently SNR per bit) is greater that -1.59 dB. These studies demonstrate that asymptotically orthogonal signaling is optimally energy efficient and highly resilient to fading even when the receiver performs hard-decision detection. However, these asymptotical performances require operation in the infinite bandwidth regime (when $M \to \infty$) or signaling with unbounded peak-to-average power ratio, i.e., vanishing duty cycle.

In this paper, motivated by practical considerations and limitations, we analyze the non-asymptotic energy efficiency of orthogonal signaling. We consider M-ary FSK with finite M.

Additionally, we investigate the energy efficiency when FSK is combined with on-off keying (or equivalently PPM) whose duty cycle is small but nonzero. On-off FSK (OOFSK) modulation introduces peakedness in both time and frequency, and provides improvements in energy efficiency over on-off keying only or FSK only.

II. CHANNEL MODEL

We consider the following channel model

$$\mathbf{r}_k = h_k \mathbf{s}_{x_k} + \mathbf{n}_k \quad k = 1, 2, 3 \ldots \tag{1}$$

where x_k is the discrete input, $\mathbf{s}_{x_k}$ is the transmitted signal when the input is x_k, and $\mathbf{r}_k$ is the received signal during the kth symbol duration. h_k is the channel gain. h_k is a fixed constant in unfaded AWGN channels, while in flat fading channels, h_k denotes the fading coefficient. $\{\mathbf{n}_k\}$ is a sequence of independent and identically distributed (i.i.d.) zero-mean circularly symmetric Gaussian random vectors with covariance matrix $E\{\mathbf{n}\mathbf{n}^\dagger\} = N_0 \mathbf{I}$ where $\mathbf{I}$ denotes the identity matrix. We assume that the system has an average energy constraint of $E\{\|\mathbf{s}_{x_k}\|^2\} \leq \mathcal{E} \quad \forall k$.

If M-ary orthogonal FSK modulation is used for transmission, then $x_k \in \{1, 2, \ldots, M\}$ and the transmitted signal has an M complex-dimensional vector representation. If $x_k = m$, $\mathbf{s}_{x_k} = \mathbf{s}_m = (s_{m,1}, s_{m,2}, \ldots, s_{m,M})$ where $s_{m,m} = \sqrt{\mathcal{E}} e^{j\theta_m}$ and $s_{m,i} = 0$ for all $i \neq m$. The phases θ_m can be arbitrary. The received signal $\mathbf{r}_k$ and noise $\mathbf{n}_k$ are also M dimensional. We assume that the receiver quantizes the received vector $\mathbf{r}_k$ by performing energy detection.

III. ENERGY EFFICIENCY OF FSK MODULATION

In this section, we assume that the transmitter employs FSK modulation and the receiver performs energy detection. In the well-known noncoherent detection of FSK signals, $\mathbf{s}_i$ is declared as the detected signal if the ith component of the received vector $\mathbf{r}$ has the largest energy, i.e., $|r_i|^2 > |r_j|^2 \quad \forall j \neq i$. Note that this decision rule is the maximum likelihood decision rule in AWGN, coherent fading, and noncoherent Rician fading channels [9], [13], [14]. The output of the detector is denoted by $y \in \{1, 2, \ldots, M\}$. Note that with energy detection, the channel can be now regarded as a symmetric discrete channel with M inputs and M outputs.

Initially, we consider the AWGN channel and assume $h_k = 1 \forall k$. The capacity which is achieved by equiprobable FSK signals is given by[2]

$$C_M(\text{SNR}) = \log M + \sum_{l=1}^{M} P_{l,1} \log P_{l,1} \tag{2}$$

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[2] Throughout the paper, log is used to denote the logarithm to the base e i.e., the natural logarithm.

where $\text{SNR} = \frac{\mathcal{E}}{N_0}$ and $P_{l,1} = P(y = l | x = 1)$ is the probability that $y = l$ given that $x = 1$. Using the results on the error probabilities of noncoherent detection of FSK signals (see e.g., [14]), we have

$$P_{1,1} = \sum_{n=0}^{M-1} \frac{(-1)^n}{n+1} \binom{M-1}{n} e^{-\frac{n}{n+1}\text{SNR}} \text{ and } P_{l,1} = \frac{1-P_{1,1}}{M-1}$$

for $l \neq 1$. Hence, the capacity can also be expressed as

$$C_M(\text{SNR}) = \log M + P_{1,1}\log P_{1,1} + (1 - P_{11})\log\frac{1-P_{1,1}}{M-1}. \quad (3)$$

Next, we provide the behavior of the capacity in the low-SNR regime.

Proposition 1: The first derivative at zero SNR of the capacity $C_M(\text{SNR})$ in (2) is $\dot{C}_M(0) = 0$. Therefore, the bit energy required at zero spectral efficiency is

$$\left.\frac{E_b}{N_0}\right|_{\mathsf{C}=0} = \lim_{\text{SNR}\to 0}\frac{\text{SNR}\log 2}{C_M(\text{SNR})} = \frac{\log 2}{\dot{C}_M(0)} = \infty \quad \forall M \geq 2. \quad (4)$$

Proof: From (3), we can write

$$\dot{C}_M(0) = \dot{P}_{1,1}(\text{SNR}=0)(1 + \log P_{1,1}(\text{SNR}=0)) - \dot{P}_{1,1}(\text{SNR}=0)$$
$$- \dot{P}_{1,1}(\text{SNR}=0)\log\frac{1-P_{1,1}(\text{SNR}=0)}{M-1} = 0$$

Above, $\dot{P}_{1,1}(\text{SNR}=0)$ denotes the derivative of the transition probability $P_{1,1}$ with respect to SNR at $\text{SNR}=0$ while $P_{1,1}(\text{SNR}=0)$ is the value of the transition probability at $\text{SNR}=0$. The result is obtained by noting that $P_{1,1}(\text{SNR}=0) = 1/M$. $\qquad\square$

Although FSK is energy efficient asymptotically as $M \to \infty$, Proposition 1 shows that for finite M, operating at very low SNR levels is extremely energy inefficient as the bit energy requirement increases without bound with decreasing SNR regardless of how large M is. As a result, the minimum bit energy is achieved at a nonzero spectral efficiency[3], the value of which can be found through numerical analysis. In [10], the capacity and cutoff rate of M-ary FSK is studied in the AWGN, coherent Rayleigh, and noncoherent Rician fading channels and it is noted through numerical results that there exists an optimal code rate for which the required bit energy is minimized. Here, we prove that observation for any finite value of M. We note that energy inefficiency at low SNRs is not a consequence of hard decision detection. It is shown in [12] that the first derivative of the FSK capacity is zero even if soft detection is employed.

Fig. 1 plots the bit energy E_b/N_0 curves as a function of spectral efficiency for M-ary FSK in the AWGN channel for different values of M. In all cases, we observe that the minimum bit energy is achieved at a nonzero spectral efficiency C^*, and the bit energy requirements increase to infinity as spectral efficiency decreases to zero. Hence, operation below C^* should be avoided. Another observation is that the minimum bit energy and the spectral efficiency value at which the minimum is achieved decrease with increasing M. For instance, when $M = 2$, the minimum bit energy is 7.821 dB and is achieved at $\mathsf{C}^* = 0.251$ bits/s/Hz. On the other hand, when $M = 48$, the minimum
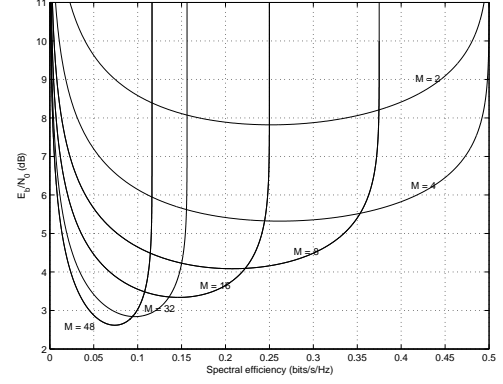
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[3]If FSK signals have a symbol duration of T, the bandwidth requirement is $\frac{M}{T}$ and the spectral efficiency is defined as $\mathsf{C}\left(\frac{E_b}{N_0}\right) = \frac{\frac{C_M(\text{SNR})\log_2 e}{T}}{\frac{M}{T}} = \frac{C_M(\text{SNR})\log_2 e}{M}$ bits/s/Hz.

Fig. 1. Bit energy E_b/N_0 vs. Spectral efficiency $\mathsf{C}(E_b/N_0)$ for energy-detected M-ary FSK in the AWGN channel. $M = 2, 4, 8, 16, 32, 48$.

bit energy has decreased to 2.617 dB and is now attained at $\mathsf{C}^* = 0.074$ bits/s/Hz. On the other hand, as M increases, the minimum bit energy is achieved at a higher SNR value. Indeed, we can show that

$$\lim_{\substack{\epsilon\to 0 \\ M\to\infty}}\left.\frac{C_M(\text{SNR})}{\text{SNR}}\right|_{\text{SNR}=(1+\epsilon)\log M} = \lim_{\substack{\epsilon\to 0 \\ M\to\infty}}\frac{C_M((1+\epsilon)\log M)}{(1+\epsilon)\log M}$$
$$= \lim_{\epsilon\to 0}\frac{1}{1+\epsilon}\lim_{M\to\infty}P_{1,1} = 1. \quad (5)$$

Hence, if SNR grows logarithmically with increasing M, the bit energy $\frac{E_b}{N_0} = \frac{\text{SNR}\log 2}{C_M(\text{SNR})}$ approaches $\log 2 = -1.59$ dB. The proof of (5) is omitted because Turin [2] has already shown that -1.59 dB is achieved if the signal duration increases as $\log M$, which in turn increases the SNR logarithmically in M.

Next, we consider fading channels. In coherent fading channels where only the receiver has perfect knowledge of the fading coefficients, the average capacity is

$$C_{M,c}(\text{SNR}) = \log M + \sum_{l=1}^{M} E_h\{P_{l,1,h}\log P_{l,1,h}\} \quad (6)$$

where $P_{1,1} = \sum_{n=0}^{M-1}\frac{(-1)^n}{n+1}\binom{M-1}{n}e^{-\frac{n}{n+1}|h|^2\text{SNR}}$ and $P_{l,1} = \frac{1-P_{1,1}}{M-1}$ for $l \neq 1$. We also consider noncoherent Rician fading channels where neither the transmitter nor the receiver knows the fading coefficients. In this case, we assume that $\{h_k\}$ are i.i.d. complex Gaussian random variables with mean $E\{h_k\} = d$ and variance $E\{|h_k - d|^2\} = \gamma^2$. In the noncoherent Rician fading channel, the capacity of M-ary FSK modulation is

$$C_{M,nc}(\text{SNR}) = \log M + \sum_{l=1}^{M} P_{l,1}\log P_{l,1} \quad (7)$$

where $P_{1,1} = \sum_{n=0}^{M-1}\frac{(-1)^n}{n(1+\gamma^2\text{SNR})+1}\binom{M-1}{n}e^{-\frac{n}{n(1+\gamma^2\text{SNR})+1}|d|^2\text{SNR}}$

and $P_{l,1} = \frac{1-P_{1,1}}{M-1}$ for $l \neq 1$. We note that these transition probabilities are obtained from the error probability expressions of FSK modulation in noncoherent Rician fading channels (see e.g., [9], [13]). Since the presence of fading unknown at the transmitter only decreases the performance, we immediately have the following corollary to Proposition 1.
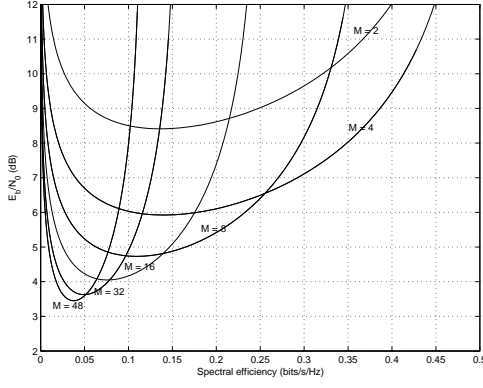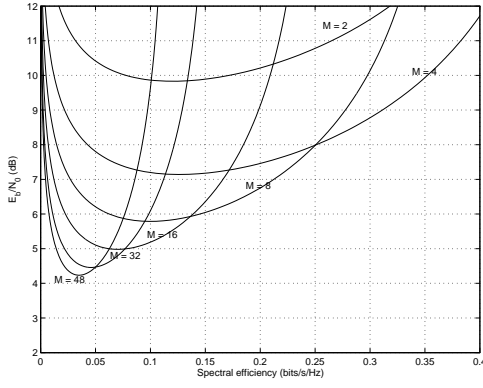
Fig. 2. Bit energy E_b/N_0 vs. Spectral efficiency $\mathsf{C}(E_b/N_0)$ for energy-detected M-ary FSK in the coherent Rician fading channel with Rician factor $\mathsf{K} = 1$.

Fig. 3. Bit energy E_b/N_0 vs. Spectral efficiency $\mathsf{C}(E_b/N_0)$ for energy-detected M-ary FSK in the noncoherent Rician fading channel with Rician factor $\mathsf{K} = 1$.

Corollary 1: The first derivatives at zero SNR of the capacities $C_{M,c}(\mathrm{SNR})$ and $C_{M,nc}(\mathrm{SNR})$ are equal to zero, i.e., $\dot{C}_{M,c}(0) = \dot{C}_{M,nc}(0) = 0$. Therefore, the bit energy required at zero spectral efficiency is infinite in both coherent and noncoherent fading channels, i.e., $\left.\frac{E_{b,c}}{N_0}\right|_{\mathsf{C}=0} = \left.\frac{E_{b,nc}}{N_0}\right|_{\mathsf{C}=0} = \infty$.

Figures 2 and 3 plot the bit energy curves for M-ary FSK transmission over coherent and noncoherent Rician fading channels. As predicted, the bit energy levels for all values of M increase without bound as the spectral efficiency decreases to zero. Due to the presence of fading, the minimum bit energies have increased with respect to those achieved in the AWGN channel. For instance, when $M = 48$, the minimum bit energies are now $E_b/N_{0\min} = 3.45$ dB in the coherent Rician fading channel and $E_b/N_{0\min} = 4.23$ dB in the noncoherent Rician fading channel. We again observe that the minimum bit energy decreases with increasing M. Fig. 4 provides the minimum bit energy values as a function of M in the AWGN and noncoherent Rician fading channels with different Rician factors. In all cases, the minimum bit energy decreases with increasing M. However, Fig. 4 indicates that approaching -1.59 dB is very slow and demanding in M. In this figure, we also note the energy penalty due to the presence of unknown fading. But, as the Rician factor K increases, the noncoherent Rician channel approaches to the AWGN channel and so do the minimum bit energy requirements.
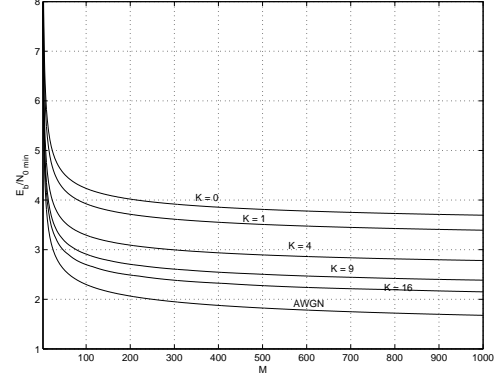

Fig. 4. Minimum bit energy $E_b/N_{0\min}$ vs. M for M-ary FSK in the AWGN channel and noncoherent Rician fading channels with Rician factors $\mathsf{K} = 0, 1, 4, 9, 16$. Note that $K = 0$ corresponds to the noncoherent Rayleigh channel.
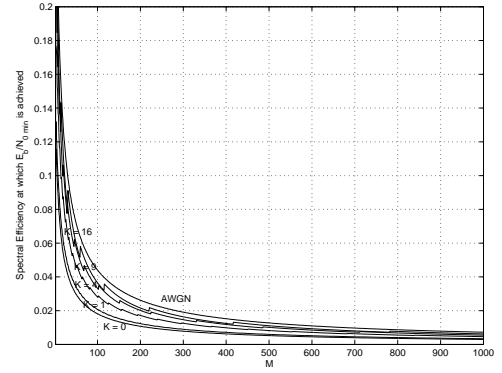

Fig. 5. Spectral efficiency at which $E_b/N_{0\min}$ is achieved vs. M for M-ary FSK in the AWGN channel and noncoherent Rician fading channels with Rician factors $\mathsf{K} = 0, 1, 4, 9, 16$.
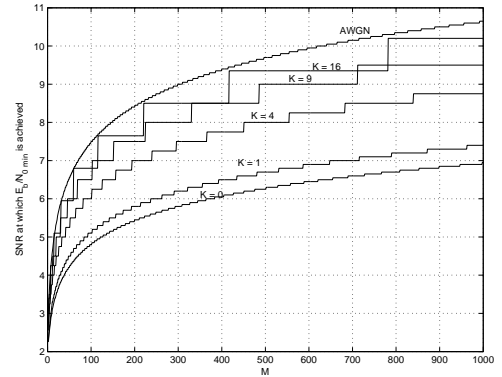

Fig. 6. SNR at which $E_b/N_{0\min}$ is achieved vs. M for M-ary FSK in the AWGN channel and noncoherent Rician fading channels with Rician factors $\mathsf{K} = 0, 1, 4, 9, 16$.

Figures 5 and 6 plot the spectral efficiencies and average received SNR values at which $E_b/N_{0\min}$ is achieved as a function of M. As also observed in Figs. 1 and 3, $E_b/N_{0\min}$ is achieved at lower spectral efficiencies as M increases. From Fig. 5, we note that the required spectral efficiencies are lower and hence the bandwidths requirements are higher in noncoherent fading channels. In Fig. 6, we see that the SNR levels at which $E_b/N_{0\min}$ is achieved increases with increasing M. As predicted in (5), the increase is logarithmic in M in the AWGN channel. Similar rates of increase are also noted for noncoherent fading channels.

IV. ENERGY EFFICIENCY OF OOFSK MODULATION

In the previous section, we have seen that the minimum bit energy decreases as the number of orthogonal frequencies, M, increases. Another technique to improve the energy efficiency is to increase the peakedness of the signal in the time domain. In [12], we have introduced the on-off frequency-shift keying (OOFSK) modulation by overlaying frequency-shift keying on on-off keying. In M-ary OOFSK modulation, the transmitter either sends no signal with probability $1 - \nu$ or sends one of M orthogonal FSK signals each with probability ν/M. In this setting, $\nu \in (0, 1]$ can be seen as the duty cycle of the transmission. While the FSK signals have energy $\mathcal{E}/\nu$, the average energy of OOFSK modulation is $\mathcal{E}$. Hence, the peak-to-average power ratio of OOFSK is $1/\nu$. No transmission is denoted by $\mathbf{s}_0 = (0, 0, \ldots, 0)$. The FSK signals have the following geometric representations:

$$\mathbf{s}_m = (s_{m,1}, s_{m,2}, \ldots, s_{m,M}) \quad m = 1, 2, \ldots, M, \quad (8)$$

where $s_{m,m} = \sqrt{\mathcal{E}/\nu}\, e^{j\theta_m}$ and $s_{m,i} = 0$ for $i \neq m$. Note that in M-ary OOFSK modulation, we have $M + 1$ possible input signals including the no signal transmission. We again assume that the received signal is hard-decision detected at the receiver. In [13], maximum a posteriori probability (MAP) detection rule for OOFSK modulation is identified and the error probability expressions are obtained. The optimal detection rule is given by the following: $\mathbf{s}_i$ for $i \neq 0$ is detected if $|r_i|^2 > |r_j|^2 \ \forall j \neq i$ and $|r_i|^2 > \tau$ where $\tau = \begin{cases} \frac{[I_0^{-1}(\xi)]^2}{4\alpha^2} & \xi \geq 1 \\ 0 & \xi < 1 \end{cases}$, $\xi = \frac{M(1-\nu)\,e^{\alpha^2}}{\nu}$, and $\alpha^2 = \frac{\text{SNR}}{\nu}$. No transmission and hence $\mathbf{s}_0$ is detected if $|r_i|^2 < \tau \ \forall i$. After hard-decision detection, the channel can be regarded as a discrete channel with $M + 1$ inputs and $M + 1$ outputs. From the error probability analysis in [13], we have the following expressions for the transition probabilities in the AWGN channel:

$$P_{0,0} = (1 - e^{-\tau})^M, \quad P_{l,0} = \frac{1}{M}(1 - (1 - e^{-\tau})^M) \quad (9)$$

$$P_{l,l} = \sum_{n=0}^{M-1} \frac{(-1)^n}{n+1} \binom{M-1}{n} e^{-\frac{n}{n+1}\alpha^2} Q_1\left(\sqrt{\frac{2}{n+1}}\,\alpha, \sqrt{2(n+1)\tau}\right) \quad (10)$$

$$P_{0,l} = (1 - e^{-\tau})^{M-1}\left(1 - Q_1\left(\sqrt{2}\,\alpha, \sqrt{2\tau}\right)\right) \quad (11)$$

for $l = 1, 2, \ldots, M$, and

$$P_{l,m} = \frac{1}{M-1}(1 - P_{m,m} - P_{0,m}) \ \forall l, m \neq 0, \text{ and } l \neq m. \quad (12)$$

In the above expressions, $Q_1(\cdot, \cdot)$ is the Marcum Q-function [8], and I_0^{-1} is the functional inverse of the zeroth order modified Bessel function of the first kind. The rate achieved by the M-ary OOFSK modulation with duty cycle ν and equiprobable FSK signals is

$$I_M(\text{SNR}, \nu) = H(y) - H(y|x) \quad (13)$$

$$= -\left((1 - \nu)P_{0,0} + \nu P_{0,1}\right)\log\left((1 - \nu)P_{0,0} + \nu P_{0,1}\right)$$

$$- M\left((1 - \nu)P_{1,0} + \frac{\nu}{M}P_{1,1} + \frac{(M-1)\nu}{M}P_{1,2}\right)$$

$$\times \log\left((1 - \nu)P_{1,0} + \frac{\nu}{M}P_{1,1} + \frac{(M-1)\nu}{M}P_{1,2}\right)$$

$$+ (1 - \nu)\left(P_{0,0}\log P_{0,0} + M P_{1,0}\log P_{1,0}\right)$$

$$+ \nu\left(P_{0,1}\log P_{0,1} + P_{1,1}\log P_{1,1} + (M-1)P_{2,1}\log P_{2,1}\right). \quad (14)$$

It is shown in [12] that in the AWGN channel, the capacity with soft detection of OOFSK signaling has a zero slope at SNR $= 0$. Since hard-decision detection decreases the capacity, we can immediately conclude that $\dot{I}_M(0, \nu) = 0$ for any fixed nonzero value of ν, and hence the bit energy required at zero spectral efficiency is still infinite. On the other hand, we know from [1] and [7] that if the duty cycle ν vanishes simultaneously with SNR, the minimum bit energy of -1.59 dB can be achieved. The following result identifies the rate at which ν should decrease as SNR gets smaller.

Theorem 1: Assume that $\nu = \frac{\text{SNR}}{(1+\epsilon)\log\frac{1}{\text{SNR}}}$ for SNR < 1 and for some $\epsilon > 0$. Then, we have

$$\lim_{\epsilon \to 0}\lim_{\text{SNR} \to 0}\frac{I_M(\text{SNR}, \nu)}{\text{SNR}} = 1 \quad (15)$$

and hence

$$\lim_{\epsilon \to 0}\lim_{\text{SNR} \to 0}\frac{\text{SNR}\log 2}{I_M(\text{SNR}, \nu)} = \log 2 = -1.59 \text{ dB}. \quad (16)$$

Proof: Note that as SNR $\to 0$, $\nu \to 0$ and $\alpha^2 = \frac{\text{SNR}}{\nu} = (1 + \epsilon)\log\frac{1}{\text{SNR}} \to \infty$. It can also be seen that $\xi \to \infty$ and $\tau \to \infty$ as SNR diminishes. From (9), we immediately note that $P_{0,0} \to 1$ and $P_{l,0} \to 0$ for $l = 2, \ldots, M$. In (10), all the terms in the summation other than for $n = 0$ vanishes because $\alpha^2 \to \infty$. Therefore, in order for $P_{l,l}$ for $l = 1, \ldots, M$ to approach 1, we need $Q_1(\sqrt{2}\,\alpha, \sqrt{2\tau}) \to 1$. Also note that if $Q_1(\sqrt{2}\,\alpha, \sqrt{2\tau}) \to 1$, then we can observe from (11) and (12) that $P_{0,l} \to 0$ and $P_{l,m} \to 0$. Hence, eventually all crossover error probabilities will vanish and correct detection probabilities will be 1.

In [8], it is shown that $Q_1(a, a\zeta) \geq 1 - \frac{\zeta}{1-\zeta}e^{-\frac{a^2(1-\zeta)^2}{2}}$ $0 \leq \zeta < 1$. From this lower bound we can immediately see that $\lim_{\text{SNR} \to 0} Q_1(\sqrt{2}\,\alpha, \sqrt{2\tau}) = 1$ if $\lim_{\text{SNR} \to 0}\frac{\tau}{\alpha^2} < 1$. Note that both α^2 and τ grow without bound as SNR $\to 0$. Recall that $\tau = \frac{[I_0^{-1}(\xi)]^2}{4\alpha^2}$. Equivalently, we have $I_0(\sqrt{4\alpha^2\tau}) = \xi = \frac{M(1-\nu)\,e^{\alpha^2}}{\nu}$. Using the asymptotic form $I_0(x) = \frac{1}{\sqrt{2\pi x}}\,e^x + \mathcal{O}\left(\frac{1}{x^{3/2}}\right)$ [16] for large x, we can easily show that $\lim_{\text{SNR} \to 0}\frac{\tau}{\alpha^2} = \left(\frac{1+\epsilon/2}{1+\epsilon}\right)^2 < 1 \ \forall \epsilon > 0$ if $\nu = \frac{\text{SNR}}{(1+\epsilon)\log\frac{1}{\text{SNR}}}$. Therefore, if ν decays at this rate, the error probabilities go to zero. It can then be shown that $\lim_{\text{SNR} \to 0}\frac{I_M(\text{SNR}, \nu)}{\text{SNR}} = \frac{1}{1+\epsilon}$. Since results hold for any $\epsilon > 0$, letting $\epsilon \to 0$ gives the desired result. $\square$

We note that Zheng *et al.* have shown in [11] that the low SNR capacity of unknown Rayleigh fading channel can be approached by on-off keying if $\frac{\log \frac{1}{\text{SNR}}}{\log \log \frac{1}{\text{SNR}}} \leq \alpha^2 \leq \log \frac{1}{\text{SNR}}$. We see a similar behavior here when FSK signals are sent over the AWGN channel and envelope detected.

In coherent fading channels where the receiver has perfect knowledge of the fading coefficients, the transition probabilities are the same as those in (9)-(12) with the only difference that we now have $\alpha^2 = \frac{\text{SNR}}{\nu}|h|^2$. As a result, the achievable rates $I_M(\text{SNR}, \nu, |h|^2)$ are also dependent on the fading coefficients and average achievable rates are obtained by finding the expected value $E_{|h|^2}\{I_M(\text{SNR}, \nu, |h|^2)\}$.

In noncoherent Rician fading channels, the transition probabilities [13] are

$$P_{0,0} = (1 - e^{-\tau})^M, \quad P_{l,0} = \frac{1}{M}(1 - (1 - e^{-\tau})^M) \quad (17)$$

$$P_{l,l} = \sum_{n=0}^{M-1} (-1)^n \binom{M-1}{n} \frac{e^{-\frac{n\alpha^2|d|^2}{n(1+\gamma^2\alpha^2)+1}}}{n(1+\gamma^2\alpha^2)+1}$$

$$\times Q_1\left(\sqrt{\frac{2\alpha^2|d|^2}{(1+\gamma^2\alpha^2)(n(1+\gamma^2\alpha^2)+1)}}, \sqrt{\frac{2(n(1+\gamma^2\alpha^2)+1)\tau}{(1+\gamma^2\alpha^2)}}\right)$$

$$P_{0,l} = (1 - e^{-\tau})^{M-1}\left(1 - Q_1\left(\sqrt{\frac{2\alpha^2|d|^2}{1+\gamma^2\alpha^2}}, \sqrt{\frac{2\tau}{1+\gamma^2\alpha^2}}\right)\right)$$

$$(18)$$

for $l = 1, 2, \ldots, M$, and

$$P_{l,m} = \frac{1}{M-1}(1 - P_{m,m} - P_{0,m}) \, \forall l, m \neq 0, \text{and } l \neq m. \quad (19)$$

In the above expressions, $\tau = \begin{cases} \Phi^{-1}(\xi) & \xi \geq 1 \\ 0 & \xi < 1 \end{cases}$, $\Phi(x) = e^{\frac{\alpha^2\gamma^2 x}{1+\alpha^2\gamma^2}} I_0\left(\frac{2\sqrt{x\,\alpha^2|d|^2}}{1+\alpha^2\gamma^2}\right)$, and $\xi = \frac{M(1-\nu)}{\nu}(1 + \alpha^2\gamma^2) e^{\frac{\alpha^2|d|^2}{1+\alpha^2\gamma^2}}$. The achievable rates can be obtained from (14). If $|d| \geq 1$, then following the same steps as in the proof of Theorem 1, we can show that the minimum bit energy of -1.59 dB is achieved as SNR $\to 0$ if $\nu = \frac{\text{SNR}}{(1+\epsilon)\log\frac{1}{\text{SNR}}}$.

Figs. 7 and 8 plot the bit energies as a function of spectral efficiency of 8-OOFSK with different duty cycle factors in the AWGN and noncoherent Rayleigh fading channels. We immediately observe that decreasing the duty cycle lowers the minimum bit energy. Hence, increasing the signal peakedness in the time domain improves the energy efficiency. In the AWGN channel, while regular 8-FSK (8-OOFSK with $\nu = 1$) has $E_b/N_{0\min} = 4.08$ dB, 8-OOFSK with $\nu = 0.01$ has $E_b/N_{0\min} = 2.017$ dB. However, this energy gain is obtained at the cost of increased peak-to-average ratio. We also note that unknown fading again induces an energy penalty with respect to the AWGN channel as observed by comparing Figs. 7 and 8.

Fig. 7. Bit energy E_b/N_0 vs. Spectral efficiency $\mathsf{C}(E_b/N_0)$ for 8-OOFSK in the AWGN channel. The duty cycle values are $\nu = 1, 0.8, 0.5, 0.3, 0, 1, 0.01$ and 0.001.

Fig. 8. Bit energy E_b/N_0 vs. Spectral efficiency $\mathsf{C}(E_b/N_0)$ for 8-OOFSK in the noncoherent Rayleigh fading channel. The duty cycle values are $\nu = 1, 0.8, 0.5, 0.3, 0, 1, 0.01$ and 0.001.

REFERENCES

[1] M. J. E. Golay, "Note on the theoretical efficiency of information reception with PPM," *Proc. IRE*, vol. 37, pp. 1031, Sept. 1949.

[2] G. L. Turin, "The asymptotic behavior of ideal M-ary systems," *Proc. IRE*, vol. 47, pp. 93-94, Jan. 1959.

[3] I. Jacobs, "The asymptotic behavior of incoherent M-ary communications systems," *Proc. IEEE*, vol. 51, pp. 251-252, Jan. 1963.
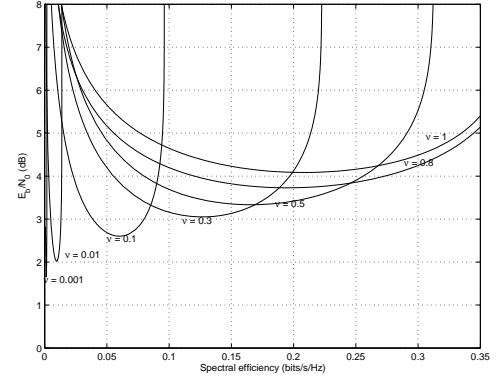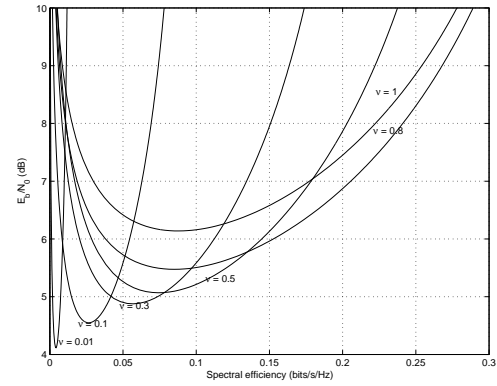
[4] J. N. Pierce, "Ultimate performance of M-ary transmissions on fading channels," *IEEE Trans. Inform. Theory*, vol. IT-12, pp. 2-5, Jan. 1966.

[5] A. Lapidoth and S. Shamai (Shitz), "Fading channels: How perfect need 'perfect side information' be?," *IEEE Trans. Inform. Theory*, vol. 48, pp. 1118-1134, May 2002.

[6] S. Verdú, "On channel capacity per unit cost," *IEEE Trans. Inform. Theory*, vol. 36, pp. 1019-1030, Sep. 1990

[7] S. Verdú, "Spectral efficiency in the wideband regime," *IEEE Trans. Inform. Theory*, vol. 48, pp. 1319-1343, June 2002.

[8] M. K. Simon and M.-S. Alouni, "A unified approach to the performance analysis of digital communication over generalized fading channels", *Proc. of the IEEE*, vol. 86, no. 9, pp. 1860-1877, Sept. 1998

[9] W. C. Lindsey, "Error probabilities for Rician fading multichannel reception of binary and N-ary Signals", *IEEE Trans. Inform. Theory*, vol.10, pp. 339-350, Oct. 1964.

[10] W. E. Stark, "Capacity and cutoff rate of noncoherent FSK with nonselective Rician fading," *IEEE Trans. Commun*, vol. 33, pp. 1153-1159, Nov. 1985.

[11] L. Zheng, D. N. C. Tse, and M. Médard "Channel coherence in the low SNR regime," *IEEE Trans. Inform. Theory*, vol. 53, pp. 976-997, March 2007.

[12] M. C. Gursoy, H. V. Poor, S. Verdú, "On-Off frequency-shift keying for wideband fading channels" *EURASIP Journal on Wireless Communications and Networking*, 2006.

[13] M. C. Gursoy, "Error rate analysis for peaky signaling over fading channels," submitted to the IEEE Transactions on Communications, 2007 (available at http://www.ee.unl.edu/faculty-staff/gursoy.shtml).

[14] J. G. Proakis, *Digital Communications*. New York: McGraw-Hill, 1995.

[15] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: Wiley, 1991.

[16] E. Butkov, *Mathematical Physics*. Addison-Wesley, 1968.